Physics 201 Name KEY

Exam 2 – Electrodynamics

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April 19, 2010

This is a closed book examination but during the exam you may refer to a 4"x6" note card with words of wisdom you have written on it. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on all questions.

A general reminder about problem solving:

- Focus
  - Draw a picture of the problem
  - What is the question? What do you want to know?
  - List known and unknown quantities
  - List assumptions
- Physics
  - Determine approach – What physics principles will you use?
  - Pick a coordinate system
  - Simplify picture to a schematic (if needed)
- Plan
  - Divide problem into sub-problems
  - Modify schematic and coordinate system (if needed)
  - Write general equations
- Execute
  - Write equations with variables
  - Do you have sufficient equations to determine your unknowns?
  - Simplify and solve
- Evaluate
  - Check units
  - Why is answer reasonable?
  - Check limiting cases!
- Show All Your Work!

1) [4 PTS] A generator with an effective (rms) voltage of 1.5 V is connected to a transformer on a side with 1000 windings. The other side has only 10 windings so the effective (rms) output voltage is
   a) 150 V
   b) 15 V
   c) 0 V
   d) 150 mV
   e) 15 mV

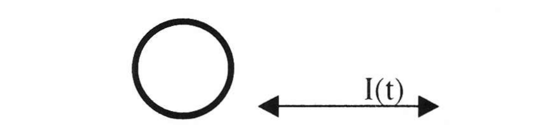
$V_{emf} = -N\frac{d\Phi_m}{dt}$ so $\frac{V_1}{N_1} = \frac{V_2}{N_2}$ $\qquad \frac{1.5\text{V}}{1000} = \frac{V_2}{10}$

Need an AC voltage

$\frac{15\text{V}}{1000} = V_2$

$15\text{mV} = V_2$

2) [4 PTS] A closed loop is placed next to a wire. The wire carries an rms current of 50 mA. The loop does not move relative to the wire.
   a) There will be no induced current.
   b) There will only be an induced current if the loop moves parallel to the wire.
   c) There will only be an induced current if the loop is rotated 90° so its surface normal vector is parallel to the wire.
   d) There is a constant induced current.
   e) There is an oscillating induced current.
   f) None of the above



$\vec{B}$ field wraps around the wire

$B \propto I$ so since I(t) oscillates the magnetic field oscillates.

$\Phi_m = \int B \cdot dA$ ~~and~~ and $\frac{d}{dt}\Phi_m \neq 0$ ⇒ at times (+) and then (−), so induced emf (voltage) will oscillate

3) [4 PTS] When the effective (rms) voltage and current in an ac circuit are in-phase, we know

a) the capacitive reactance is zero ×
b) the inductive reactance is zero ×
c) the impedance is zero ×
d) the total reactance is ½ of the resistance ×
✓ e) the circuit is being operated at its resonant frequency
f) both (a) and (b) ×
g) both (c) and (e) ×

In phase means the circuit is operating at resonant frequency ($\delta = 0$)

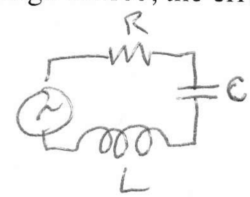
$\omega_0 = \frac{1}{(LC)^{1/2}}$ $Z = (R^2 + X^2)^{1/2}$

when $\omega = \omega_0$ $Z = R$, $X = 0$

$\delta = 0 = \tan^{-1}\frac{X}{R}$ so $X = X_L - X_C = 0$

4) [4 PTS] An inductor, capacitor and resistor are connected in series to an AC voltage source. If you double the frequency of the voltage source, the effect on the circuit is to

a) double the capacitive reactance. ×
b) double the inductive reactance. ✓
c) leave the total reactance unchanged. ×
d) halve the inductive reactance. ×
e) halve the impedance. ×
f) none of the above.



$X_C = \frac{1}{\omega C}$ $X_C' = \frac{1}{2} X_C$

$X_L = \omega L$ $X_L' = 2X_L$

$Z = (R^2 + (X_L - X_C)^2)^{1/2}$

$X = X_L - X_C$

5) [4 PTS] The more rapidly a magnet moves away from a copper ring, the

a) lower the induced current in the ring.
b) greater the inductance of the ring.
c) the lower the inductance of the ring.
d) greater the induced current in the ring
e) none of the above

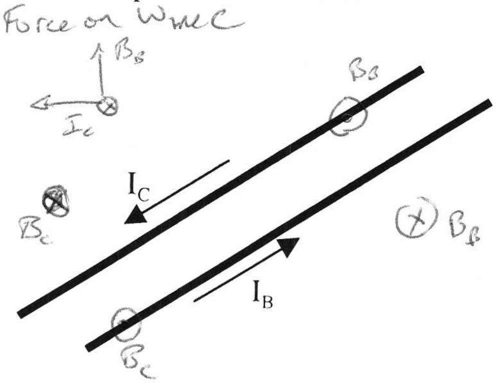
$\frac{d\Phi_m}{dt}$ increases so V_{emf} increases

$V = IR$ so current increases

6) [4 PT] Two very long wires, 60 cm apart, are hung parallel to each other. Current flows down each wire in opposite directions. Wire C has a current of ¼ Amps and wire B has a current of ½ Amp.

a) The two wires are attracted $F_C = ¼F_B$
b) The two wires are attracted $F_C = ½F_B$
c) The two wires are attracted $F_C = F_B$
d) The two wires are repelled $F_C = F_B$
e) The two wires are repelled $F_C = 2F_B$
f) The two wires are repelled $F_C = 4F_B$

Force on wire C

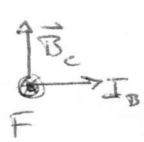


We now $\vec{F}_C = -\vec{F}_B$

$\vec{F} = q\vec{v} \times \vec{B}$

The force on each wire is towards the outside

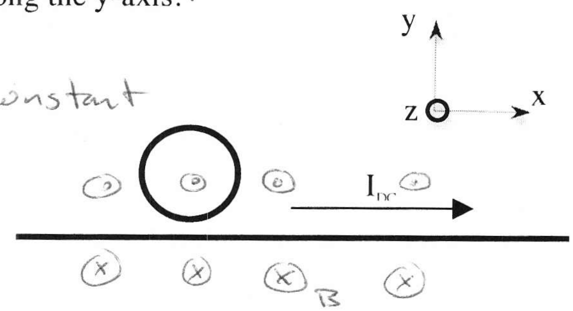
Force on wire B



7) [4 PTS] A closed loop is placed next to a wire. The wire carries a constant current of 50 mA. The loop is started rotating next to the wire with a constant angular velocity ω.

a) There will be no induced current. x
b) There is a constant induced current. x
c) There will only be an induced current if ω is along the z-axis. x
d) There is an oscillating induced current if ω is along the y-axis. ✓
e) None of the above x

$\vec{B}$ wraps around the wire and is constant



$\frac{d\Phi_m}{dt} = -V_{emf}$ $\Phi_m = \int \vec{B} \cdot d\vec{A}$

while $\vec{B}$ is constant the area will change if the loop rotates around $\hat{y}$ or $\hat{x}$

8) [15 PT] You have connected an inductor (L=8.0 mH), a capacitor (C=80μF) and resistor (R=100 Ω) in series. You connect your LCR circuit to a function generator that is producing a sinusoidal voltage signal with a peak to peak amplitude of 16 volts at a frequency of 880 Hz.

a) What is the resonant frequency for this circuit?
b) Write the equation for the voltage across the function generator if $V_0(t=0\text{ sec}) = 0$ V.
c) What is the impedance of this LCR circuit when it is at resonance?
d) What is the RMS current passing through the resistor?
e) What is the voltage as a function of time across the capacitor?

Useful mathematical relationships:

$\sin^2(\theta) + \cos^2(\theta) = 1$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

$\int_b^c \frac{a}{x} = a\ln x\Big|_b^c = a[\ln c - \ln b] = a\ln\left(\frac{c}{b}\right)$

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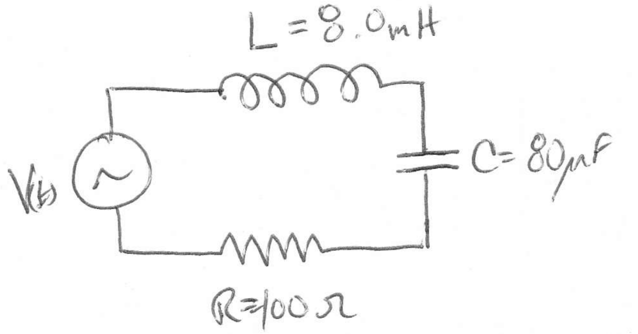

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

$V(t) = V_p \sin(\omega t + \delta)$

$\delta = 0$

$\omega = 2\pi f$

$V_p = 16V/2 = 8V$

$f = 880 Hz$

(a) $\omega_0 = \frac{1}{(LC)^{1/2}} = 1250 \text{ rad/sec}$

$\omega_0 = 2\pi f_0$ so $f_0 = \frac{\omega_0}{2\pi} = 199 Hz$

(b) $V_0(t) = V_p \sin(\omega t + \delta)$

$V_0(t=0) = V_p \sin(\omega 0 + \delta)$ so $\delta = 0$ so sine function is $\emptyset$

$V_0(t) = 8 \sin(2\pi \, 880 Hz \, t)$ Volts

$= 8 \sin(5530 t)$ Volts

(c) $Z = (R^2 + (X_L - X_C)^2)^{1/2}$ when $\omega = \omega_0$ $X_L = X_C$

$Z = R$ when $\omega \equiv \omega_0$ (at resonance)

(d) $V_R = IR$ $\quad I = \frac{V_0}{Z}$ $\quad V_0 = \frac{V_p}{\sqrt{2}} = 5.66 V_{rms}$

$Z = \left(100^2 + \left(5530 \cdot 8 mH - \frac{1}{5530 \cdot 80 \mu F}\right)^2\right)^{1/2}$

$= 108 \Omega$

$I_{rms} = \frac{5.66 V_{rms}}{108 \Omega} = 0.052 A_{rms}$

(e) $V_c(t) = ?$

$Q = CV_c$

$I = C\frac{dV_c}{dt}$ so $V_c(t) = \frac{1}{C}\int I\,dt$

so need to find $I(t)$

$I(t) = \frac{V_0(t)}{Z}$

$\therefore I_p = \frac{V_p}{|Z|} = 0.074$ Amps

$\delta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{42}{100}\right) = 22.8^\circ$

$X = X_L - X_C = 42\,\Omega$

$V_c(t) = \frac{1}{C}\int 0.074\sin(\omega t + 22.8^\circ)\,dt$

$= \frac{-1}{\omega C}\cdot 0.074\cdot\cos(\omega t + 22.8^\circ)$

voltage

$V_c(t) = \frac{-V_p}{\omega C Z}\cos(\omega t + \delta)$

$V_c(t) = -0.17\cos(5530\,t + 23^\circ)$ Volts